LOGIC

What is logic ?

- The word logic derived from the Greek word ‘logos’, Which means reason
- Thus logic deals with method of reasoning.
- The study of logic helps in increasing ones ability of systematic & logical reasoning & develops the skill of understanding validity of statements.

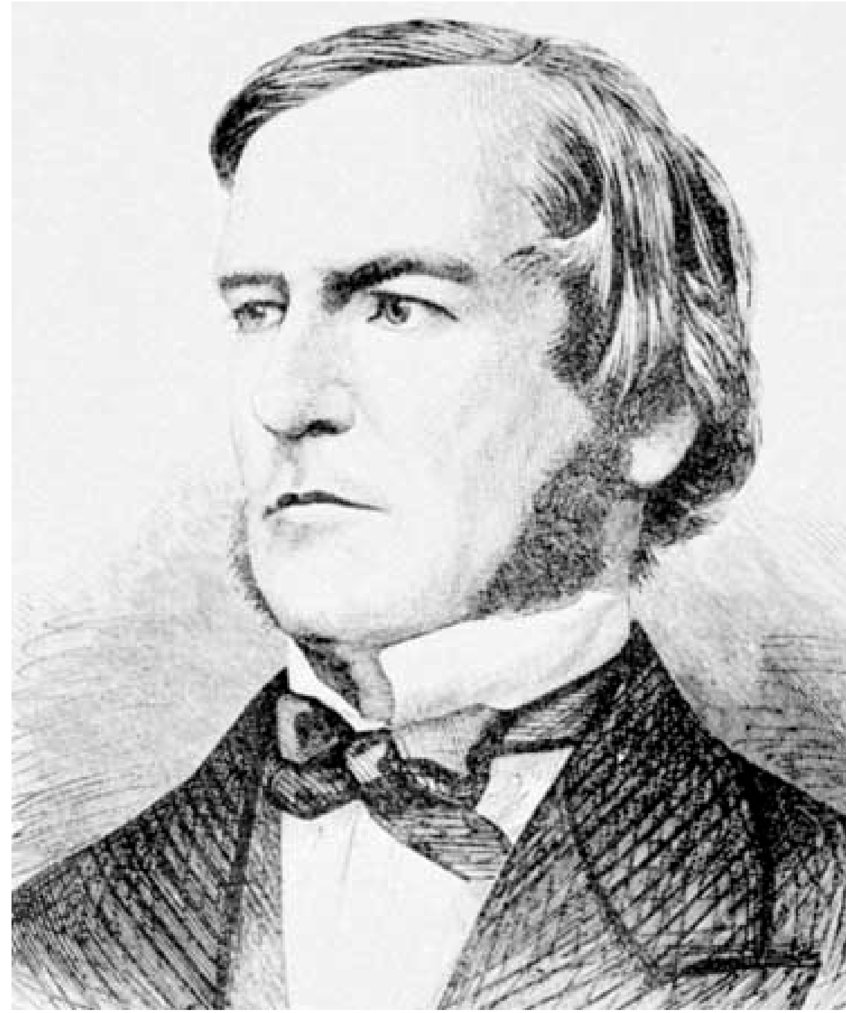
history

- **Aristotle (382-322 B.C.)**
- the great philosopher & thinker laid the foundation of study of logic in systematic form.

- **<u>George Boole</u>**

<u>(1815-1864)</u>

- The axiomatic approach to logic was first propounded by English philosopher & mathematician George Boole.
- Hence it is called as Boolean Algebra or Boolean logic.



What is Statement ?

- A proposition or Statement is a declarative sentence which is either true or false but not both simultaneously.
- Sentences which are imperative ,exclamatory or interrogative are not statement.

(i) Ice floats in water.
(ii) China is in Europe.
(iii) $2 + 2 = 4$
(iv) $2 + 2 = 5$
(v) Where are you going?
(vi) Do your homework.

The first four are propositions, the last two are not.
Also, (i) and (iii) are true, but (ii) and (iv) are false.

Logical connectives ,compound statements &truth table

- The words or group of words like AND,OR,NOT,IF…..THEN,IF AND ONLY IF,….. Can be used to join two simple statements.
- Such words or group of words are called as Logical connectives.
- The statements formed by combining 2 or more than two simple statements by using the logical connectives are called compound statements.

We shall discuss the diff connectives given in the following table

Sr.No.	Connectives	symbol	Term used to describe the compound statement
1	And	^	Conjunction
2	Or	v	Disjunction
3	Not	~	Negation
4	If….then	→	Conditional or implication
5	..if & only if…or iff..	↔	Bi-conditional or double implication

Conjunction(^)

p	q	P ^ q
T	T	T
T	F	F
F	T	F
F	F	F

- **Thus p^q is defined to have truth value “true” if both p &q have the truth values “true”.**

Disjunction (v)

p	q	P v q
T	T	T
T	F	T
F	T	T
F	F	F

- **The disjunction pvq is false when both p & q are false.**

Negation (~)

p	~ p
T	F
F	T

- **The connective 'not ' operates only one statement**

Implication (→)

P	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

- **p →q is defined to have the trurh value ‘false’ if p has truth value ‘true’ & q has truth value ‘false’.**

DOUBLE IMPLICATION (↔)

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

- **p ↔q is defined to have the truth value ‘true’ if p & q both have same truth values (‘true’ or ‘false’) otherwise p ↔q is defined to have the truth values ‘false’.**

- Let p be the statement "DATAENDFLAG is off,"
- q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentences in symbolic notation.

1) DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.

2) DATAENDFLAG is off but ERROR is not equal to 0.

3) DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.

4) DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.

SOLUTION:

P : DATAENDFLAG is off

q : ERROR equals 0

r : SUM is less than 1,000

1) (P v q) ^ r

2) P ^ (~ q)

3) P ^ (~ q v r)

4) (~ p ^ q) v r

- Write negations for each of the following statements.

P : Jim is tall and Jim is thin

~p : Jim is not tall or Jim is not thin

Q : $-1 < x \leq 4$.

~Q : $-1 \nless x$ or $x \lneqq 4$

P : John is 6 feet tall and he weighs at least 200 pounds

~P : John is not 6 feet tall or he weighs less than 200 pounds

A **tautology** is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables.

A statement whose form is a tautology is a tautological statement.

A **contradiction** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradiction is a contradictory statement.

- Given any statement variables p,q, and r, a tautology t and a contradiction c, the following logical equivalences hold.

1. Commutative laws	:	$p\wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws	:	$(p\wedge q)\wedge r \equiv p\wedge(q \wedge r)$	$(p\vee q)\vee r \equiv p\vee(q \vee r)$
3. Distributive laws	:	$p\wedge(q \vee r) \equiv (p\wedge q)\vee(p\wedge r)$	
		$p\vee(q \wedge r) \equiv (p\vee q)\wedge(p\vee r)$	
4. Identity laws	:	$p\wedge t\equiv p$	$p \vee c\equiv p$
5. Negation laws	:	$p\vee \sim p \equiv t$	$p\wedge \sim p \equiv c$
6. Double negative law	:	$\sim(\sim p) \equiv p$	
7. Idempotent laws	:	$p\wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound laws	:	$p\vee t\equiv t$	$p\wedge c\equiv c$
9. De Morgan's laws	:	$\sim(p\wedge q) \equiv \sim p \vee \sim q$	$\sim(p\vee q) \equiv \sim p\wedge \sim q$
10. Absorption laws	:	$p\vee(p\wedge q) \equiv p$	$p \wedge(p\vee q) \equiv p$
11. Negations of t and c	:	$\sim t\equiv c$	$\sim c\equiv t$

Let h =“John is healthy,”

w =“John is wealthy,” and

s = “John is wise.”

a. John is healthy and wealthy but not wise.
b. John is not wealthy but he is healthy and wise.
c. John is neither healthy, wealthy, nor wise.
d. John is neither wealthy nor wise, but he is healthy.
e. John is wealthy, but he is not both healthy and wise.

The **contrapositive** of a conditional statement of the form

“If p then q” is If ∼q then ∼p.

Symbolically, The contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$.

If today is Easter, then tomorrow is Monday.

CONTRAPOSTIVE: If tomorrow is not Monday, then today is not Easter

Suppose a conditional statement of the form “If p then q” is given.

1. The converse is “If q then p.”
2. The inverse is “If∼p then∼q.”
3. Symbolically,

   The converse of $p \rightarrow q$ is $q \rightarrow p$, and

   The inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$.

- An **argument** is a sequence of statements,

and an **argument form** is a sequence of statement forms.

- All statements in an argument and all statement forms in an argument form, except for the final one, are called **premises** (or assumptions or hypotheses).

- The final statement or statement form is called the **conclusion.**

- The symbol ∴, which is read “therefore,” is normally placed just before the conclusion.

// * To say that an argument form is valid means that no matter what particular statements are substituted for the statement variables in its premises *//

if the resulting premises are all true, then the conclusion is also true. To say that an argument is valid means that its form is valid.

An argument form consisting of two premises and a conclusion is called a **syllogism**.

The first and second premises are called the **major premise** and **minor premise,** respectively.

The term modus ponens is Latin meaning "method of affirming" (the conclusion is an affirmation).

The most famous form of syllogism in logic is called modus ponens.

It has the following form: If p then q.

p ∴ q

Here is an argument of this form:

If the sum of the digits of 371,487 is divisible by 3, then 371,487 is divisible by 3.

The sum of the digits of 371,487 is divisible by 3.

∴ 371,487is divisible by 3.

A valid argument form called modus tollens. If it has the following form:
If p then q.

$\sim q$

$\therefore \sim p$

Here is an example of modus tollens: If Zeus is human, then Zeus is mortal.

Zeus is not mortal.

∴ Zeus is not human.

If 870,232 is divisible by 6, then it is divisible by 3.

870,232 is not divisible by 3.

∴ 870, 232 is not divisible by 3

Modus Ponens	$p \rightarrow q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \rightarrow c$ $\therefore p$

In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a–e below) and challenged the reader to use them to figure out the location of the treasure.

a. If this house is next to a lake, then the treasure is not in the kitchen.
b. If the tree in the front yard is an elm, then the treasure is in the kitchen.
c. This house is next to a lake.
d. The tree in the front yard is an elm or the treasure is buried under the flagpole.
e. If the tree in the back yard is an oak, then the treasure is in the garage.

Where is the treasure hidden?

Modus Ponens	$p \to q$ p $\therefore q$	Elimination	a. $p \lor q$ $\sim q$ $\therefore p$ b. $p \lor q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	Transitivity	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	a. p $\therefore p \lor q$ b. q $\therefore p \lor q$	Proof by Division into Cases	$p \lor q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	a. $p \land q$ $\therefore p$ b. $p \land q$ $\therefore q$		
Conjunction	p q $\therefore p \land q$	Contradiction Rule	$\sim p \to c$ $\therefore p$

You are about to leave for school in the morning and discover that you don't have your glasses. You know the following statements are true:

a. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

b. If my glasses are on the kitchen table, then I saw them at breakfast.

c. I did not see my glasses at breakfast.

d. I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.

e. If I was reading the newspaper in the living room then my glasses are on the coffee table.

Where are the glasses?

Solution:

P: Reading News paper in the Kitchen (RK)
Q: glasses on the kitchen (GK)
R: Saw glasses at breakfast (GB)
S: Reading the news paper in living room(RL)
T: Glasses are on the coffee table.

Glasses are on the coffee table